Photo-Emission of a Single-Electron Wave-Packet in a Strong Laser Field

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The radiation emitted by a single-electron wave packet in an intense laser field is considered. A relation between the exact quantum formulation and its classical counterpart is established via the electron's Wigner function. In particular we show that the wave packet, even when it spreads to the scale of the wavelength of the driving laser field, cannot be treated as an extended classical charge distribution but rather behaves as a point-like emitter carrying information on its initial quantum state. We outline an experimental setup dedicated to put this conclusion to the test.

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The availability of super-intense lasers has stimulated interest in relativistic electron dynamics in strong driving fields [1]. Experimenters have observed the effects of ponderomotive acceleration, the Lorentz drift, and plasma wake-fields through direct detection of electrons ejected from an intense laser focus. Photoemission from relativistically driven plasmas has also been studied.

Much theory and computational effort has been devoted to the dynamics of free-electron wave packets driven by intense fields [2, 3, 4] and the associated scattered radiation [3, 5, 6]. Coherent emission from many electrons can be viewed in the forward direction with the emerging laser beam. Here, we consider incoherent photoemission by free electrons out the side of a focused laser, as a means of studying electron dynamics.

A free electron wave packet with an initial spatial size on the scale of an atom undergoes natural quantum spreading, which eventually reaches the scale of an optical wavelength, as illustrated in Fig. 1 [4]. Moreover, an electron wave packet born through field ionization is pulled from its parent atom at a finite rate, typically emerging over multiple laser cycles. This, combined with the Lorentz drift and sharp ponderomotive gradients found in a tight relativistic laser focus, can cause a single-electron wave packet to be strewn throughout a volume several laser wavelengths across [6]; different portions of the same wave packet can even be propelled out opposite sides of the laser focus.

The question naturally arises as to how a single-electron wave packet radiates, especially when it undergoes such highly non-dipole dynamics, where different parts of the electron wave packet experience entirely different phases of a stimulating laser field. So far, the problem has been treated [5, 6] within an intuitively appealing model where the quantum probability current is multiplied by the electron charge to produce an extended current distribution used as a source in Maxwell's equations [7]. The intensity computed classically from the extended current distribution is then associated with the probability of measuring a photon. Due to interference, this approach can lead to dramatic suppression of radiation for many directions. We note that semi-classical descriptions have, in general, proven very useful to understand processes like above-threshold ionization or high-harmonic generation, which arise from intense laser-matter interactions [8].

In this letter, we provide a fully quantum mechanical treatment of photoemission by a single-electron wave packet in a laser field and relate it to a classical description via the electron's Wigner function. We show that no interference occurs between emission from different parts of an initially Gaussian wave packet, even if spatially large. In a plane-wave driving field, this result holds for wave packets of any shape and size. The radiative response can be mimicked by the incoherent emission of a classical ensemble of point charges. In a focused laser beam, interferences can occur for certain initial electron states, but these interferences are of a different nature than those arising from a semi-classical current-distribution approach. We outline an experimental arrangement able to probe the single-electron emission behavior by combining methods from strong-field physics and quantum optics.

We first examine radiation interferences that arise from treating a single electron as an extended charge distribution in a semiclassical picture. Fig. 1 shows an example

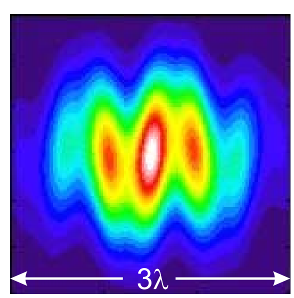

FIG. 1: An electron wave packet after natural spreading from an initially Gaussian-shaped size of 1Å. The spreading takes places during 190 cycles in a plane wave with intensity 2×10^{18} W/cm^2 and wavelength $\lambda = 800$ nm.

of an electron wave packet (probability density), computed using the Klein-Gordon equation [4]. In the low-intensity limit, a wave-packet such as shown in Fig. 1 can be associated with a classical Gaussian current distribution: $\mathbf{J} \sim \hat{z} r_0^{-3} \mathrm{e}^{-r^2/r_0^2}\, \mathrm{e}^{i\kappa x}$, where r_0 characterizes the spatial extent of the distribution with fixed overall charge. The distribution is stimulated by a plane wave with wave four-vector $\kappa = (\omega_\kappa, \boldsymbol{\kappa})$ traveling in the x-direction and polarized in the z-direction. The intensity of the Thomson scattering from the distribution is

$$I\,(\theta,\phi) \sim \sin^2\theta\, \mathrm{e}^{-|\boldsymbol{\kappa}|^2 r_0^2 (1-\sin\theta\cos\phi)}, \tag{1}$$

where $\theta = 0$ defines the z-direction, and $\theta = \pi/2$ with $\phi = 0$ defines the x-direction. Fig. 2 shows the intensity emitted into several directions, as well as the overall emission, as a function of distribution size. The forward emission does not vary from that of a single point oscillator with equal net charge. In the perpendicular direction, the intensity drops by orders of magnitude as the wave function grows to the scale of the wavelength or bigger. This leads to a substantial loss in the overall scattered-light energy [9].

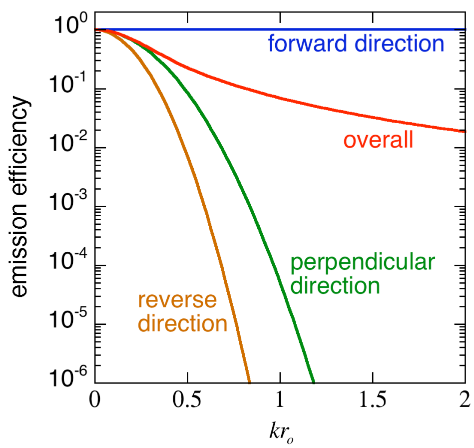

FIG. 2: Efficiency of light scattering into different directions as a function of size of a driven Gaussian charge distribution.

Measurements of Compton/Thomson scattering provide an indication that electrons do not radiate as extended charge distributions. For example, >10 keV photons scattered from electrons bound to helium corresponds to a scenario where the size of the electron wave packet is larger than the wavelengths involved. However, the cross section for the scattered photons with energy well below the electron rest energy is known [7, 10] to follow Eq. (1) with $r_0 = 0$ (with ϕ averaged over all angles for unpolarized light). It is interesting to note that A. H. Compton initially proposed a "large electron" model to explain the decrease in cross section with angle for harder x rays, which he later abandoned when the effect of momentum recoil was understood [11].

Quantum electrodynamics provides the general framework to calculate the radiation from a single-electron wave packet. The radiated intensity is proportional to the expectation value of the correlation function of the current density, rather than the expectation value of the current density of the wave packet [12]. From the definition of the spectral intensity of the emitted radiation $d\varepsilon_{\mathbf{k}} = \frac{c}{4\pi^2}\left\langle \hat{\mathbf{H}}_\omega^{(-)} \hat{\mathbf{H}}_\omega^{(+)} \right\rangle R_0^2 d\Omega d\omega$, with radiation magnetic field $\hat{\mathbf{H}}_\omega^{(\pm)} = i\mathbf{k} \times \hat{\mathbf{A}}_\omega^{(\pm)}$ and vector-potential $\hat{\mathbf{A}}_\omega^{(\pm)} = \frac{e\mathrm{e}^{ikR_0}}{cR_0} \int d^4x\, \hat{\mathbf{J}}^{(\pm)}(x)\mathrm{e}^{ikx}$, together with the current-density operators $\hat{\mathbf{J}}^{(\pm)}$ in the Heisenberg representation, one derives

$$d\varepsilon_{\mathbf{k}} = \frac{e^2}{4\pi^2 c} \int d^4x \int d^4x' \mathrm{e}^{ik(x-x')} \left\langle \left(\mathbf{k} \times \hat{\mathbf{J}}^{(-)}(x)\right) \left(\mathbf{k} \times \hat{\mathbf{J}}^{(+)}(x')\right) \right\rangle d\Omega d\omega. \tag{2}$$

Here $^{(\pm)}$ indicates the positive/negative frequency parts of operators, R_0 the distance to the observation point from the coordinate center, $\mathbf{k}$ the radiation wave-vector, $d\Omega$ the emission solid angle, e the electron charge and c the speed of light. Equation (2) can be represented in a more familiar form via the transition current density $\mathbf{J}_{p'i}(x)$ in the Schrödinger picture:

$$d\varepsilon_{\mathbf{k}} = \frac{e^2}{4\pi^2 c} \int \frac{d^3p'}{(2\pi\hbar)^3} \left| \int d^4x\ (\mathbf{k} \times \mathbf{J}_{p'i}(x))\, \mathrm{e}^{ikx} \right|^2 d\Omega d\omega \tag{3}$$

where $\mathbf{J}_{p'i}(x) = \left(\bar{\psi}_{\mathbf{p}'}(x) \boldsymbol{\gamma} \psi_i^{(\mathrm{L})}(x)\right)$, $\psi_i^{(\mathrm{L})}(x)$ is the initial electron wave packet in the laser field, $\psi_{\mathbf{p}'}(x)$ is a complete set of free electron states, and $\boldsymbol{\gamma}$ are the Dirac matrices. Equation (3) indicates that the total probability of photon emission should be calculated as an incoherent sum over the final momentum states of the electron, even though in the experiment the final electron momentum could be undetected.

Although Eq. (3) shows the general way to calculate the emission intensity, it is difficult to apply in a real experimental situation, as the quantum eigenstates of the electron in a focused laser beam are usually unknown. It is therefore indeed desirable to mimic the quantum electrodynamical result of Eq. (3) by means of classical electrodynamical calculations (in the quasi-classical limit, when the photon energy is much smaller than the electron rest energy and recoil effects are negligible). We demonstrate how to do this by way of the example of one-photon Thomson scattering in a focused laser beam. Choosing the initial wave packet in the form of $\psi_i(x) = \int d^3p\, \alpha(\mathbf{p}) \psi_{\mathbf{p}}(x)$, one can express the quantum-mechanical formula for spectral intensity of Eq.(3) via the Wigner function $\rho_{\mathrm{w}}(\mathbf{r},\mathbf{p}) = \int d^3q\, \alpha(\mathbf{p}+\mathbf{q}/2)\, \alpha^*(\mathbf{p}-\mathbf{q}/2)\mathrm{e}^{i\mathbf{qr}}$ of the initial electron wave packet as follows:

$$d\varepsilon_{\mathbf{k}\lambda} = \frac{e^2\omega^2}{4\pi^2 c^3} \int d^3r \int d^3p\, \rho_{\mathrm{w}}(\mathbf{r},\mathbf{p}) M_{\mathbf{k}\lambda}(\mathbf{r},\mathbf{p})\, d\Omega d\omega, \tag{4}$$

where $M_{\mathbf{k}\lambda}(\mathbf{r},\mathbf{p}) = (2\pi\hbar)^2 \int d^3\kappa \int d^3\kappa' e^{i(\boldsymbol{\kappa}-\boldsymbol{\kappa}')\boldsymbol{r}} \times A_0(\boldsymbol{\kappa}) A_0^*(\boldsymbol{\kappa}') \Im_\lambda(\mathbf{p}_+,\mathbf{p}',\boldsymbol{\kappa},\mathbf{k}) \Im_\lambda^*(\mathbf{p}_-,\mathbf{p}',\boldsymbol{\kappa}',\mathbf{k}) \delta(\varepsilon_{p_+} + \hbar\omega_\kappa - \varepsilon_{p'} - \hbar\omega)\, \delta(\varepsilon_{p_-} + \hbar\omega_{\kappa'} - \varepsilon_{p'} - \hbar\omega)$ gives the radiation by an electron of momentum $\mathbf{p}$; $\Im_\lambda(\mathbf{p},\mathbf{p}',\boldsymbol{\kappa},\mathbf{k})$ is defined via $\int d^4x\, e^{ikx}(\mathbf{e}_\lambda^* \mathbf{J}_{p'p}) = (2\pi\hbar)^4 \int d^3\kappa A_0(\boldsymbol{\kappa}) \Im_\lambda(\mathbf{p},\mathbf{p}',\boldsymbol{\kappa},\mathbf{k}) \delta^{(4)}(p+\hbar\kappa-p'-\hbar k)$, $\varepsilon_p = c\sqrt{\mathbf{p}^2+m^2c^2}$, $\mathbf{p}_\pm = \mathbf{p} \pm \hbar(\boldsymbol{\kappa}'-\boldsymbol{\kappa})/2$, $\mathbf{p}' = \mathbf{p} + \hbar(\boldsymbol{\kappa}'+\boldsymbol{\kappa})/2 - \mathbf{k}$, $\mathbf{e}_\lambda$ is the polarization of the emitted photon, and $A_0(\boldsymbol{\kappa})$ the Fourier component of the focused laser beam. When the Wigner function is non-negative (e.g., for a Gaussian wave packet), it may be interpreted as the initial electron distribution in phase space. The message of the structure of Eq. (4) is that the total photoemission probability is an *incoherent* sum over the contributions of each local phase-space element of the electron distribution. If, for example, the phase-space distribution consists of two separate parts: $\rho_{\rm w} = \rho_{\rm w}^{(1)} + \rho_{\rm w}^{(2)}$, then the intensities emitted from each part incoherently add up to yield the total radiation intensity. In the quasi-classical limit, the term $M_{\mathbf{k}\lambda} \approx |\mathcal{M}_{\mathbf{k}\lambda}(\mathbf{r},\mathbf{p})|^2$, with $\mathcal{M}_{\mathbf{k}\lambda}(\mathbf{r},\mathbf{p}) = 2\pi\hbar \int d^3\kappa\, e^{i\boldsymbol{\kappa}\boldsymbol{r}} A_0(\boldsymbol{\kappa}) \Im_\lambda(\mathbf{p},\mathbf{p}',\boldsymbol{\kappa},\mathbf{k})\, \delta(\varepsilon_p + \hbar\omega_\kappa - \varepsilon_{p'} - \hbar\omega)$ can be directly related to the classical electrodynamical calculation: in the nonrelativistic limit $\Im_\lambda(\mathbf{p},\mathbf{p}',\boldsymbol{\kappa},\mathbf{k}) = (\mathbf{e}_\lambda^* \mathbf{e}_\kappa)$ ($\mathbf{e}_\kappa$ is the polarization of the $\boldsymbol{\kappa}$-component of the driving field) and Eq. (4) describes the incoherent *average* of the radiation intensity over the initial electron probability distribution in phase space. The radiation can thus be modelled by a classical ensemble of point emitters, taken individually.

A different situation arises when the Wigner function is negative in some phase-space region, indicating intrinsic quantum behavior. As an example, we consider an electron in a superposition of two momentum states $|\mathbf{p}_1>$ and $|\mathbf{p}_2>$; in this case interference in the emitted radiation is possible. In fact, the final electron state $|\mathbf{p}'>$ with emission of a photon of certain momentum $\mathbf{k}$ can be reached by two indistinguishable paths [13]: either from the state $|\mathbf{p}_1>$ or from $|\mathbf{p}_2>$ by absorption of different photons $\boldsymbol{\kappa}_{1,2} = \mathbf{k} + \mathbf{p}' - \mathbf{p}_{1,2}$ from the external field, giving rise to interference. This effect is included in Eq. (4) which cannot be modelled by classical means here. In the case of a Gaussian wave packet interferences are suppressed by the continuous spectrum of initial electron momenta which give rise to many interfering paths whose contributions largely cancel out. In any case, interferences do not occur in a plane-wave driving field due to its uniform propagation direction $\boldsymbol{\kappa}$. Then Eq. (4) reduces to the incoherent superposition $d\varepsilon_{\mathbf{k}\lambda} = \int d^3p\, |\alpha(\mathbf{p})|^2 \varepsilon_{\mathbf{k}\lambda}(\mathbf{p}) d\Omega d\omega$ of the spectral energies $\varepsilon_{\mathbf{k}\lambda}(\mathbf{p}) = \frac{e^2\omega^2}{4\pi^2c^3} M_{\mathbf{k}\lambda}(0,\mathbf{p})$ radiated by an electron of momentum $\mathbf{p}$ [14].

The quantum interference effects above have to be clearly distinguished, though, from those arising in the *coherent* part of the radiation mentioned in the introduction. The latter is calculated employing an ensemble average of the current operator $<\mathbf{J}(x)>$ as a source for the expectation value of the radiated field $<\mathbf{E}>$. In that picture, the spectral component of the coherent radiation intensity $|<\mathbf{E}_\mathbf{k}>|^2$ involves interference of the fields in the classical sense; i.e., all transitions with emission of a photon of certain momentum $\mathbf{k}$ interfere, independent of the final electron state. In the example of two superimposed momentum states, these are the transitions $|\mathbf{p}_i> \to |\mathbf{p}_j>$ with $i,j \in \{1,2\}$. The coherent radiation is analogous to the classical radiation of a modulated charge distribution here.

The coherent radiation is qualitatively different from the total radiation in Eq. (4). It accounts for the radiated field averaged over the quantum ensemble, which excludes the incoherent field with a fluctuating phase [15]. In the case of a single electron, the coherent radiation is only a small part of the total radiation, but becomes dominant in the case of $N \gg 1$ radiating electrons which simultaneously interact with an applied field. As in the case of radiation from an N-atom ensemble [12], the intensity of the phase-matched coherent radiation is multiplied by a factor of $N(N-1)$, while the incoherent radiation is multiplied by N, which becomes negligible. For example, any experiment on high-harmonic generation from (many) atoms measures the coherent emission (see, e.g., [16]). We note that the terminology of a "single-atom response" that is commonly used in this context, is therefore misleading.

Finally, we show that the radiation scattered from a single free electron in a laser is detectable by modern experimental techniques. The feasibility of seeing scattered light depends crucially on whether the electron wave packet radiates with the strength of a classical point-like electron, as argued above. If one looks at light near the frequency of the stimulating light (linear Thomson scattering), the rate of emitted photons is proportional to the stimulating intensity. Thus, beam fluence rather than peak intensity determines the number of photons scattered by an electron in a laser. Nevertheless, relativistic intensities may be helpful for spectral-discrimination purposes: Because the Lorentz drift pushes the electron in the forward direction of the laser, the scattered fundamental light is typically red shifted from ~ 800 nm to longer wavelengths, which is a signature that could be distinguished with bandpass filters and detected with an avalanche photodiode.

We envision an electron born in the laser field (e.g. the second electron pulled from He at $\sim 2 \times 10^{16}$ W/cm^2). Electrons given up by atoms during the leading edge of the pulse tend to be pushed out of the focus by the ponderomotive gradient. The density of donor atoms can be chosen such that on average one electron experiences the highest intensities (10^{-7} torr–10^{-5} torr, depending on the focal volume). Free electrons might also be prepared using a suitable pre pulse. Only an electron experienc-

ing the highest intensity receives a substantial forward drift and the accompanying red shift for emission in the direction perpendicular to laser propagation. Keying in on this red-shift signature may be critical for differentiating authentic scattered photon events from other noise sources. Fast timing in the photon-detection electronics can also suppress false signals, for example, scattered from the walls of the experimental chamber.

We computed a representative single classical electron trajectory in a tightly focused vector laser field with a peak intensity of 10^{19} W/cm^2, duration 35 fs, and wavelength 800 nm. Fig. 3(a) shows the total radiated energy in the far field emitted from the electron as it is released on the rising edge of the pulse. The electron trajectory eventually exits the side of the focus due to ponderomotive gradients. Much of the scattered radiation emerges out the side of the laser focus. The total radiated energy (all angles and frequencies) from the single electron trajectory is only ~ 0.24 eV, indicating less than one photon per shot.

Fig. 3(b) shows the spatial distribution of light with wavelengths falling between 850 nm and 950 nm. This accounts for approximately 20% of the total emitted power. Assuming 10% collection efficiency, this would amount to an average 0.005 eV energy per shot, or one photon per 300 shots. This can of course be increased if more electrons are used, where the light radiated out the side of the focus adds incoherently for a random distribution. The observation of blue-shifted light may also afford an opportunity for discrimination against background. Intensities above 10^{19} W/cm^2 are not ideal for this experiment; a strong Lorentz drift redirects the photoemission into the far forward direction.

In conclusion, we have studied the amount of light that an electron scatters out the side of a laser focus. We have

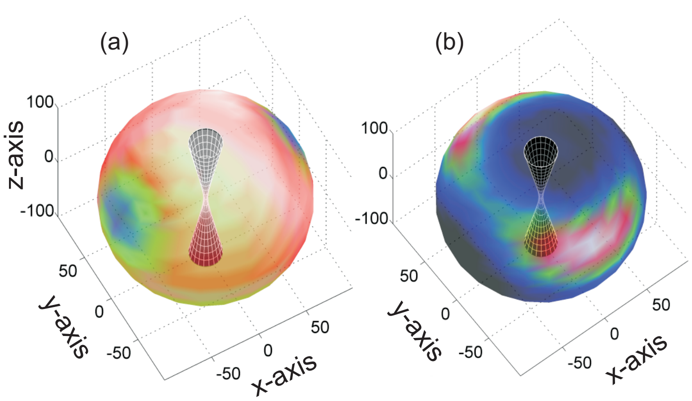

FIG. 3: (a) Far-field intensity (at 100 μm distance) of light scattered from a single electron trajectory born on axis during the early rising edge of an intense (10^{19} W/cm^2) laser pulse. Red (blue) indicates regions of high (low) intensity. The laser beam (mesh) has waist $w_0 = 3\lambda$. The total scattered energy is 0.24 eV. (b) Far-field intensity filtered to wavelengths between 850 nm and 950 nm with total scattered energy 0.05 eV.

shown that individual electrons radiate with the strength of point emitters. The electron's initial quantum state is imprinted on the radiation spectrum via its Wigner function, which in general allows for interference of different electron momentum components. The latter is qualitatively distinct from the classical interference in the coherent radiation of an extended charge distribution. Our results can be tested in an experiment that combines for the first time the sensitive techniques of quantum optics (e.g., single-photon detectors) with the traditionally opposite and incompatible discipline of high-intensity laser physics.

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